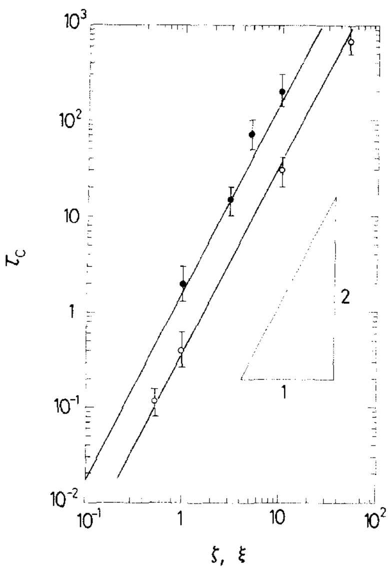

FIG. 4. Non-dimensional time to reach convective steady state as a function of ζ for forced convection (○) and ξ for natural convection (●).

It would be interesting to compute a transient heat transfer by natural convection from a vertical cylinder placed in an aquifer. Assuming the following physical properties of the aquifer and the cylinder dimensions: $\alpha = 10^{-6}$ m^2 s^{-1}, $\nu = 10^{-7}$ m^2 s^{-1}, $\sigma = 1$, $k_e = 2$ W m^{-1} °C^{-1}, $K = 10^{-10}$ m^2, $a = 0.05$ m, $L = 1$ m, $\Delta T = 10$°C, the corresponding non-dimensional parameters become $Ra_L \cong 10$, $\xi \cong 6$. The resulting average heat transfer coefficient from the cylinder at steady state is about 4.6 W m^{-2} °C^{-1}. Referring to Fig. 5, the time required to reach steady state is about 38 h.

In summary one has found that regardless of forced and natural convection the transient heat transfer from the isothermal cylinder with flows parallel to its axis is dominated by radial heat conduction during a short time. The transitional dimensionless time from the conduction to the convective steady state is proportional to ζ^2 in forced convection and to ξ^2 in natural convection, respectively. For a given cylinder height L/a a vigorous flow generally shortens the time to reach steady state, while for a given flow the greater height lengthens the time.

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Evaluation of flux models for radiative transfer in cylindrical furnaces

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1. INTRODUCTION

IN A PREVIOUS paper [1], the accuracy of several flux-type models for three-dimensional radiative heat transfer has been assessed by applying these radiation models to the prediction of distributions of the radiative flux density and the radiative energy source term of a rectangular enclosure problem and by comparing their predictions with exact solutions produced earlier by the same author [2]. The problem was based on data taken from a large-scale experimental furnace with steep temperature gradients typical of operating furnaces.

A significant number of industrial furnaces and combustors are cylindrical in shape. Therefore, it is considered necessary to evaluate the flux-type models produced earlier for cylindrical furnaces [3–5] by applying them to the prediction of radiative flux density and source term distributions of a cylindrical enclosure problem based on data reported previously on a pilot-scale experimental furnace [6] and by comparing their predictions with the exact values reported previously [7].

The radiation models to be evaluated are given below.

(1) A Schuster–Hamaker type four-flux model for an axi-symmetrical radiation field derived by Lockwood and Spalding [3].

(2) A Schuster–Schwarzschild type four-flux model derived by Siddall and Selçuk [4].

(3) A Schuster–Schwarzschild type four-flux model produced by Richter and Quack [5].

All these models had previously been employed as part of

NOMENCLATURE

K_a volumetric absorption coefficient of the medium [m^{-1}]
q component of radiative flux density vector [$W m^{-2}$]
Q source term for radiative energy [$W m^{-3}$]
R, Γ, Z coordinate directions.

Superscript
$\tilde{}$ dimensionless.

complete prediction procedures and predicted temperature and radiative heat flux distributions have been compared with experimentally determined data [3–5, 8, 9]. However, it has been found impossible to decide whether discrepancies between the predictions and measurements are attributable directly to the flux model employed or to inaccuracies in the submodels used for the prediction of flow, reaction, etc.

The use of exact solutions for testing purposes provides a means for assessing the accuracy of predictions of these radiation models in isolation from the models of flow and reaction.

In this paper, therefore, the accuracy of each of these radiation models is tested by applying it to the prediction of distributions of the radiative flux density and the radiative energy source term of a cylindrical enclosure problem and by comparing its predictions with the exact solutions reported previously [7, 10].

2. THE TEST PROBLEM

The flux-type models considered have been tested by making predictions for a black-walled enclosure problem for which exact solutions have been produced previously [7]. The enclosure problem is based on data reported by Wu and Fricker [6] on a pilot-scale experimental furnace with steep temperature gradients typically encountered in industrial furnaces.

The experimental furnace under consideration is a vertical cylinder fired from the bottom end wall with natural gas and operates under atmospheric pressure. The side walls are water cooled. A detailed description of the data obtained from the experimental furnace and used as input data for flux-type models can be found elsewhere [7, 10].

3. NUMERICAL SOLUTION PROCEDURE

The partial differential equations representing the radiation models under consideration have been re-cast into finite difference forms by using the control volume approach. As the variation of gas and wall temperatures is axisymmetrical, the enclosure has been subdivided into 2×20 control volumes in the r- and z-directions, respectively. A medium grid point lies at the geometrical centre of each control volume and a surface grid point lies in the centre of each control volume face in contact with the walls of the enclosure. Hence the total number of medium and surface grid points are 2×20 and $(2 \times 2 + 20)$, respectively. The resulting sets of simultaneous algebraic equations have then been solved by the iterative procedure developed by Peaceman and Rachford [11] for numerical solution of the algebraic equations with a coefficient matrix of the tri-diagonal type. This procedure can be described as 'forward elimination followed by backward substitution'.

4. EVALUATION OF THE FLUX MODEL PREDICTIONS

Point values of the dimensionless radiative energy source term and flux density for 2×20 medium grid points have been produced using:

(a) Lockwood and Spalding's four-flux model for plane parallel radiation—Model 1;

(b) Lockwood and Spalding's four-flux model for isotropic radiation—Model 2;

(c) Siddall and Selçuk's four-flux model for plane parallel radiation—Model 3;

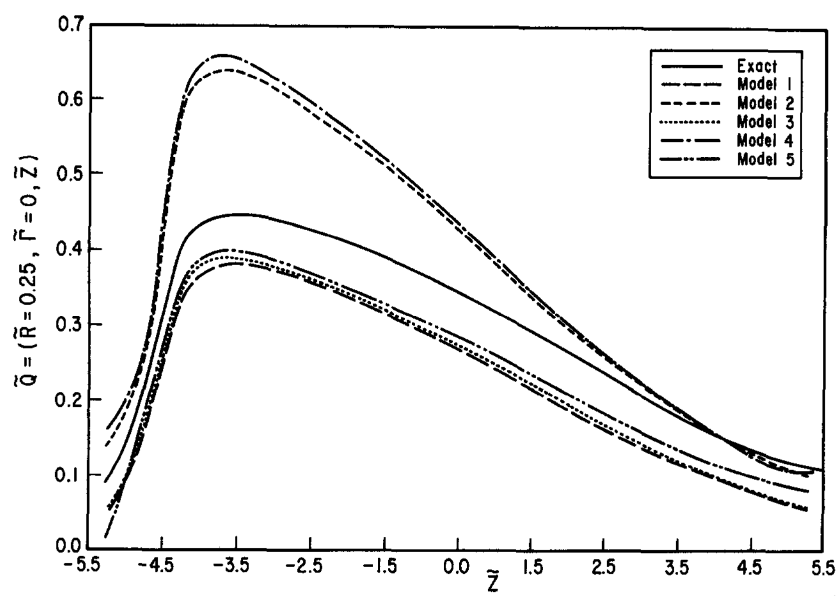

FIG. 1. Comparison between the exact values and flux model predictions of dimensionless radiative energy source terms along ($\tilde{R} = 0.25$, $\tilde{\Gamma} = 0$, $\tilde{Z}$).

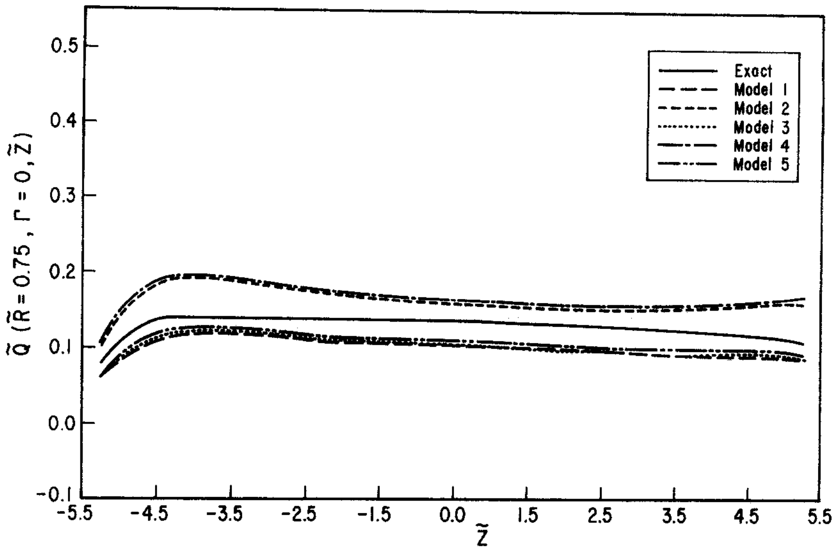

FIG. 2. Comparison between the exact values and flux model predictions of dimensionless radiative energy source terms along ($\tilde{R} = 0.75$, $\tilde{\Gamma} = 0$, $\tilde{Z}$).

(d) Siddall and Selçuk's four-flux model for isotropic radiation—Model 4;

(e) Richter and Quack's four-flux model—Model 5.

The predictions of these models have been compared with the exact solutions reported previously in the literature [7, 10].

In the discussion that follows, all physical quantities are expressed in dimensionless forms which are obtained by dividing them by the shortest dimension of the enclosure or by the maximum emissive power of the gas, depending on the quantity.

4.1. *Source term distributions*

Figure 1 shows the comparison between flux model predictions of the dimensionless source term and the exact values for points ($\tilde{R} = 0.25$, $\tilde{\Gamma} = 0$, Z). These grid points represent the points at the centre of the row of control volumes nearest to the furnace axis. It can be seen that the exact source term distribution follows the physically expected trend, rising steeply from the burner wall onwards, going through a maximum and decreasing continuously towards the exit. The maximum of the source term distribution occurs at the same location as the maximum of the temperature distribution. It can also be noted that the trend of the distributions predicted by the flux models is the same as that of the exact distribution and that the distributions are overpredicted by Models 2 and 4 and underpredicted by Models 1, 3 and 5.

Figure 2 illustrates the comparison between the exact values of the dimensionless source term and the distributions predicted by the flux models for grid points ($\tilde{R} = 0.75$, $\tilde{\Gamma} = 0$, $\tilde{Z}$). These grid points represent the medium points nearer to the side wall. It can be seen that good agreement is obtained and that the source term distributions for these grid points show smaller variation along the length of the furnace than those for other medium grid points. This is consistent with the uniform temperature distribution in the medium near the wall of the enclosure.

A condensed comparison of the flux model predictions of the dimensionless source term values is contained in Table 1. Two values are given for each model; the maximum point percentage error and the average absolute percentage error both of which give measures of the accuracy of predicted source terms.

As can be seen from Table 1, Model 5 produces more accurate results than the other models.

Table 1. Comparison of flux model predictions of dimensionless source terms

Flux model	Maximum percentage error	Average absolute percentage error
Model 1	44.98	23.38
Model 2	−52.40	24.16
Model 3	48.84	22.67
Model 4	−79.54	28.34
Model 5	74.44	18.68

4.2. *Flux density distributions*

Figure 3 illustrates the comparison between the point values of the dimensionless flux density to the side wall in the positive r-direction predicted by the flux models and exact solutions for surface grid points. It can be seen that Models 2 and 4 produce fairly good agreement and that Models 1, 3 and 5 underestimate the flux densities to the wall over the whole length of the enclosure.

A condensed comparison of the flux model predictions of the dimensionless flux densities is contained in Table 2. As can be seen from Table 2, the level of agreement decreases in the following order: Model 2, Model 4, Model 5, Model 1 and Model 3.

To provide a global check on the accuracy of the flux model predictions, the total rate of removal of radiative energy through the wall and the total rate of generation of radiative energy within the enclosed medium were calculated and compared with the exact values. Table 3 shows the errors in generated and removed radiative energy produced by the flux model predictions. It can be seen that the percentage errors in generated and removed radiative energy are almost equal for each model, implying that each model produces consistent results, although different from the exact values.

5. CONCLUSIONS

Three flux-type models for cylindrical enclosures filled with an absorbing–emitting medium of constant properties have been applied to the prediction of distributions of the radiative flux density and the energy source term of a black-walled enclosure problem. The problem is based on data reported previously on a pilot-scale experimental furnace

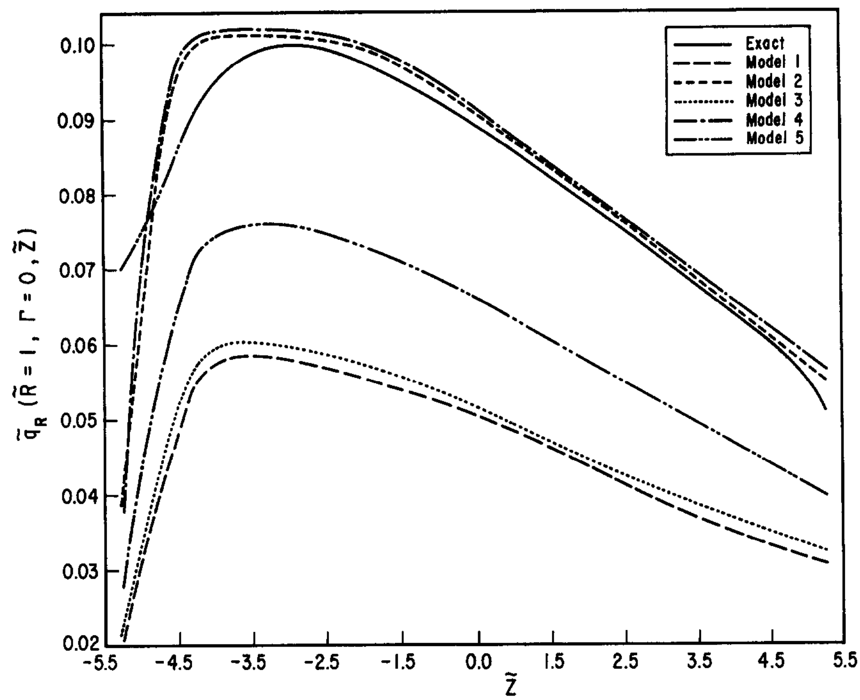

FIG. 3. Comparison between the exact values and flux model predictions of dimensionless flux densities to the side walls.

Table 2. Comparison of flux model predictions of dimensionless flux densities to the side wall

Flux model	Maximum percentage error	Average absolute percentage error
Model 1	69.20	44.24
Model 2	43.97	5.34
Model 3	70.11	43.54
Model 4	46.86	7.63
Model 5	59.37	27.07

Table 3. Comparison of flux model predictions of the percentage errors in generated and removed radiative energy

Flux model	Percentage error in generation	Percentage error in removal
Model 1	21.46	21.46
Model 2	−25.27	−25.27
Model 3	21.17	21.17
Model 4	−26.18	−26.18
Model 5	15.97	15.97

with steep temperature gradients typically encountered in industrial furnaces. The flux-type models which have been employed are a Schuster–Hamaker type model for plane parallel and isotropic radiation fields derived by Lockwood and Spalding and two Schuster–Schwarzschild type models produced by Siddall and Selçuk and Richter and Quack; the former utilizing hemispherical subdivisions for plane parallel and isotropic radiation fields and the latter utilizing conical subdivisions. The accuracy of the models have been tested by comparing their predictions with exact solutions reported earlier in the literature. On the basis of comparisons the following conclusions have been reached.

(1) The four-flux model of Richter and Quack is a reasonably satisfactory method of predicting both the radiative energy source term and flux density distributions.

(2) The four-flux models of Lockwood and Spalding and Siddall and Selçuk for an isotropic radiation field provide a useful method for predicting the flux density distributions at the walls of the enclosure. However, with regard to the distribution of the radiative energy source term, a relatively poor agreement is obtained.

(3) The four-flux methods of Lockwood and Spalding and Siddall and Selçuk for plane parallel radiation are not found satisfactory.

(4) Predictive ability of each model relative to the exact solutions are expected to remain the same for non-black walled enclosures although the order of magnitude of the accuracies might change.

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